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The use of multibody simulation programs for the study of large displacement motions of systems of flexible bodies adds another dimension to the model reduction problem. In general, straightforward criteria are available for performing model reduction at the system level; however, simulation codes normally require modal data input for each component body of a system rather than for the system as a whole. It is thus always necessary to determine the modes to be retained for each component based on knowledge of the system modes of interest.

The method presented in this report for component model reduction utilizes submatrices of the system modal matrix as transformation matrices used to accomplish the first phase of a two phase matrix diagonalization process. This method is systematic and, as demonstratedabove using simple mass-spring systems, the method gives very accurate results. The method has also been tested on a model of the Galileo spacecraft, and gave excellent results.

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Project Title: Component Model Reduction in Flexible Multibody Systems

Submitted by: School of Engineering and Architecture
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INTRODUCTION

Many engineering systems comprise several bodies connected together, with active control between bodies. Specific examples of such systems include robots and manipulators, space vehicles, missiles, and precision pointing systems. Because of the increasing tendency towards lightweight components, many such systems are partially or totally composed of flexible bodies. The dynamics of such systems can be studied by experimentation or analysis, or, preferably, both. When an analytical approach is used, modeling is usually one of the first issues to be addressed. In the study of a complex structure or a system of interconnected flexible bodies, most modeling strategies rely on a finite dimensional representation of each flexible component; and the smaller the dimension, the more tractable the analysis. The term model reduction, as used in structures and structural dynamics, refers to the process of replacing a complex structure possessing a large number of degrees of freedom by a relatively simple model with only a few degrees of freedom. The main constraint imposed on this process is that the resulting model should be simple enough to render the analysis at hand tractable, yet "rich" enough to retain the salient features of the original structure. Obviously, model reduction as defined above, is a crucial step in the modeling process. It has in fact received considerable attention in the literature[1-10] because of its importance in several areas of engineering, particularly in the consideration of control methodologies for aerospace systems. Many existing model reduction techniques solve the "single body problem"; that is, they start with one (generally large and complex) flexible body and end up with another very much simpler system, that retains certain properties of the original body that are relevant to the work at hand.

A different type of model reduction problem is encountered whenever one is compelled to work with components of a complex system. Such a situation may arise in the analysis of a large structure such as a space platform; here, different analysts are generally assigned different components of the structure. Hence, a reduced order model of each component must be generated. A similar situation arises when it is desired to simulate the motions of a system of interconnected, actively controlled flexible bodies, using a simulation package such as DISCOS[11] or TREETOPS[12]. These programs require that each body in a given system be characterized separately. For a rigid body, geometry and mass properties (e.g. mass, moments of inertia, mass center location, ... etc.) are sufficient. On the other hand, a flexible body necessitates that body mode shapes, frequencies, and damping characteristics be supplied to the program. And this must be done for each flexible body separately. Practical considerations drastically restrict the number of modes per body that can be retained in any given simulation. First, keeping many modes creates the problem of inputing and manipulating huge volumes of

data. Secondly, retaining high frequency modes makes the simulation expensive since integration time steps will then have to be kept very small in order to capture the contributions of these modes. Therefore, only a very limited number of modes can be accommodated in a real simulation — typically about eight modes per body. The question of modal truncation procedure at the level of system **components** thus becomes pertinent. This issue is not clearly addressed in the development of existing multibody programs. It is simply assumed that the user will somehow decide which modes (not exceeding about eight) of any given body will be used in the simulation. The most common practice is to "throw away" the modes corresponding to the highest frequencies.

PROBLEM STATEMENT

The problem to be solved in the course of the performance of this research project is really an offshoot of a bigger problem. The big problem is that of simulating the dynamics of a multibody system comprising two or more bodies connected at hinges. Some or all of the components of the system may be flexible and each hinge may permit one or more degrees of freedom between bodies. This simulation problem can be solved with the aid of one of the existing multibody simulation codes such as DISCOS[11] or TREETOPS/CONTOPS[12]. In order to use these codes, the system is usually modeled in a NASTRAN-like environment, so that mass, stiffness, and modal matrices (among other quantities) are available for the free-free vibration modes of each flexible body in the system. Component models constructed in this manner are generally too large, and must be reduced to a manageable size before they can be used in multibody simulation programs. This model reduction process cannot be arbitrary. In other words, one cannot, for example, simply decide to retain the first few low frequency modes for each body. The main task to be performed in this project is to develop a systematic and scientifically sound procedure for performing model reduction of components of such multibody systems.

MODEL REDUCTION STRATEGY

Consider a system A of n flexible bodies A_1, A_2, ..., A_n, connected by hinges, and with active control between bodies. If the system is frozen in a certain configuration, then it can be viewed as one large structure and its NASTRAN type model can be generated for this configuration. One of several existing model reduction techniques for solving single body problems can be applied to this system in order to reduce its dimension to a tractable level,

based on such criteria as control system specifications, bandwidth, etc. Thus, it is possible to determine those modes of the system as a whole that are relevant to the study at hand. Once these important system modes have been determined, the component model reduction problem postulated earlier reduces to that of finding the component modes that contribute substantially to important system modes. These are the component modes (hopefully very few) that need to be used for the purposes of simulating the dynamics of this multibody system. Hence, the type of model reduction needed to prepare modal data for multibody simulation programs can be viewed as a two-phase process. First, a system-level model reduction is performed using one of the existing single body methods. Next, a knowledge of the retained modes of the system is utilized to define useful component modes. There has been very little attention in the literature to the second part of the process as described above. This is perhaps due to the fact that the few multibody programs capable of accommodating flexible bodies are not yet enjoying widespread use. And when they have been used, it has mostly been to solve relatively simple problems where the component modes to be retained are almost obvious from intuition.

COMPONENT MODEL REDUCTION

The component model reduction procedure developed in the course of this project is described below using a two-body example. Consider a structure S which is made up of two interconnected substructures A and B as shown in Fig. 1. Suppose that a few specific modes of S have been determined to be important for a given system configuration, and it is desired to find modes of A and B that "feed" these system modes of interest.

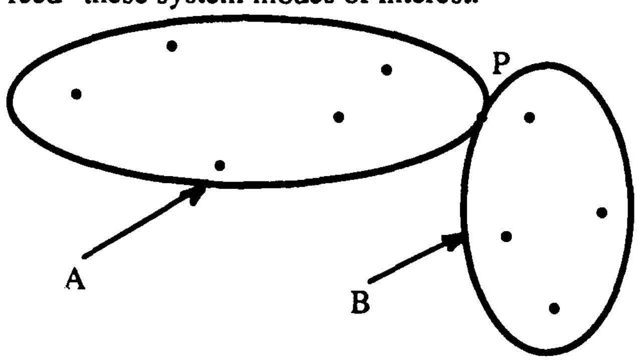

Fig.1 System of Connected Bodies

The response x_A of A to a forcing function F_A is given by the matrix differential equation

$$M_A\ddot{x}_A + K_A x_A = F_A \tag{1}$$

where M_A and K_A are the mass and stiffness matrices of A. The modal matrix ϕ_A of A can be viewed as a coordinate transformation matrix, so that

$$x_A = \phi_A y_A \tag{2}$$

and if ϕ_A is normalized with respect to M_A, as is usual in NASTRAN, Eq. (1) can be transformed into

$$I_A \ddot{y}_A + \Lambda_A y_A = \phi_A^T F_A \tag{3}$$

where I_A is an identity matrix, and Λ_A is a diagonal matrix with squares of body A modal circular frequencies as elements. If body A has n_A unrestrained degrees of freedom, then M_A, K_A, I_A, Λ_A, and ϕ_A are all square matrices of dimension n_A, while x_A, y_A, F_A are column vectors of dimension n_A. Similar equations can be written for body B assuming n_B unrestricted degrees of freedom:

$$M_B \ddot{x}_B + K_B x_B = F_B \tag{4}$$

$$x_B = \phi_B y_B \tag{5}$$

leading to

$$I_B \ddot{y}_B + \Lambda_B y_B = \phi_B^T F_B \tag{6}$$

For the complete system, S, one can also write

$$M\ddot{x} + Kx = F \tag{7}$$

$$x = \phi y \tag{8}$$

$$I\ddot{y} + \Lambda y = \phi^T F \tag{9}$$

The number of degrees of freedom n of the combined system is such that

$$n < n_A + n_B \tag{10}$$

If a vector x' is defined as

$$x' = \begin{Bmatrix} x_A \\ x_B \end{Bmatrix} = Px \tag{11}$$

where P is simply a permutation matrix which can be found by inspection, then, Eqs. (8) and (11) can be combined to give

$$x' = Px = P\phi y = \underline{\phi} y \tag{12}$$

The x' vector has dimension $n = n_A + n_B$ exactly; on the other hand, the x vector has dimension $n < n_A + n_B$ as stated earlier. If, for example, x_A and x_B represent displacements at the discrete points of A and B respectively shown in Fig. 1, and x is the displacement vector at the same points for the system viewed as one, it is easy to see that x would have fewer elements than the sum of the elements of x_A and x_B. The reason is that the displacement of a point such as P, common to both A and B, would appear in both x_A and x_B — that is twice in the x' vector, but only once in the x vector. Also, the matrix $\underline{\phi}$ is obtained from the system modal matrix ϕ by repeating some of the rows, and rearranging some others. The next step is to partition $\underline{\phi}$ as shown in Eq. (13) below, and also delete those of its columns that correspond to the system modes to be dropped.

$$\begin{Bmatrix} x_A \\ x_B \end{Bmatrix} = \begin{bmatrix} \phi_A \\ \phi_B \end{bmatrix} \{y\} \tag{13}$$

The resulting submatrices of $\underline{\phi}$ are φ_A and φ_B. The φ_A matrix will now have dimension n_A by n', just as φ_B will become a n_B by n' matrix, where n' (<n) is the number of modes retained at the system level. Defining

$$x_A = \varphi_A u_A \tag{14}$$

$$x_B = \varphi_B u_B \tag{15}$$

and substituting Eq. (14) into Eq. (1), one obtains

$$M_A \varphi_A \ddot{u}_A + K_A \varphi_A u_A = F_A \tag{16}$$

Premultiplication by $\varphi_A{}^T$ yields

$$\varphi_A^T M_A \varphi_A \ddot{u}_A + \varphi_A^T K_A \varphi_A u_A = \varphi_A^T F_A \tag{17}$$

or

$$\underline{M}_A \ddot{u}_A + \underline{K}_A u_A = \varphi_A^T F_A \tag{18}$$

Note that $\underline{M}_A$, $\underline{K}_A$ are square matrices of dimension n' by n' and they will not normally be diagonal. The eigenvalue problem corresponding to Eq. (18) is then solved to produce a modal matrix with the associated eigenvalues. In other words, if ζ_A is the modal matrix, one can use the transformation

$$u_A = \zeta_A v_A \tag{19}$$

to change Eq. (18) to

$$\zeta_A^T \underline{M}_A \zeta_A \ddot{v}_A + \zeta_A^T \underline{K}_A \zeta_A v_A = \zeta_A^T \varphi_A^T F_A \tag{20}$$

or

$$I_A \ddot{v}_A + \lambda_A v_A = \zeta_A^T \varphi_A^T F_A \tag{21}$$

It turns out that λ_A approximates a submatrix of Λ_A, and

$$\psi_A = \varphi_A \zeta_A \tag{22}$$

is a submatrix of ϕ_A. The elements of λ_A turn out to be precisely the eigenvalues of the n' modes of body A that contribute most to those modes of the system that are retained. Similar steps can be followed to produce an equation for body B that corresponds to Eq. (21):

$$I_B \ddot{v}_B + \lambda_B v_B = \zeta_B^T \varphi_B^T F_B \tag{23}$$

where λ_B again approximates a submatrix of Λ_B, and

$$\psi_B = \varphi_B \zeta_B \tag{24}$$

is also a submatrix of ϕ_B. Here again, λ_B contains the eigenvalues of the n' modes of body B that contribute strongly to the system modes of interest. Results can be checked by using any good modal synthesis scheme to reassemble the reduced bodies A and B. The result should be eigenvalues and eigenvectors that match those retained for the system initially.

EXAMPLES

To illustrate the effectiveness of the above model reduction method, consider two "bodies" A and B shown below in Figure 2. Each is simply a set of rigid disks connected by torsional springs. The given masses are in kilogram and the spring stiffnesses are in N-m/rad.

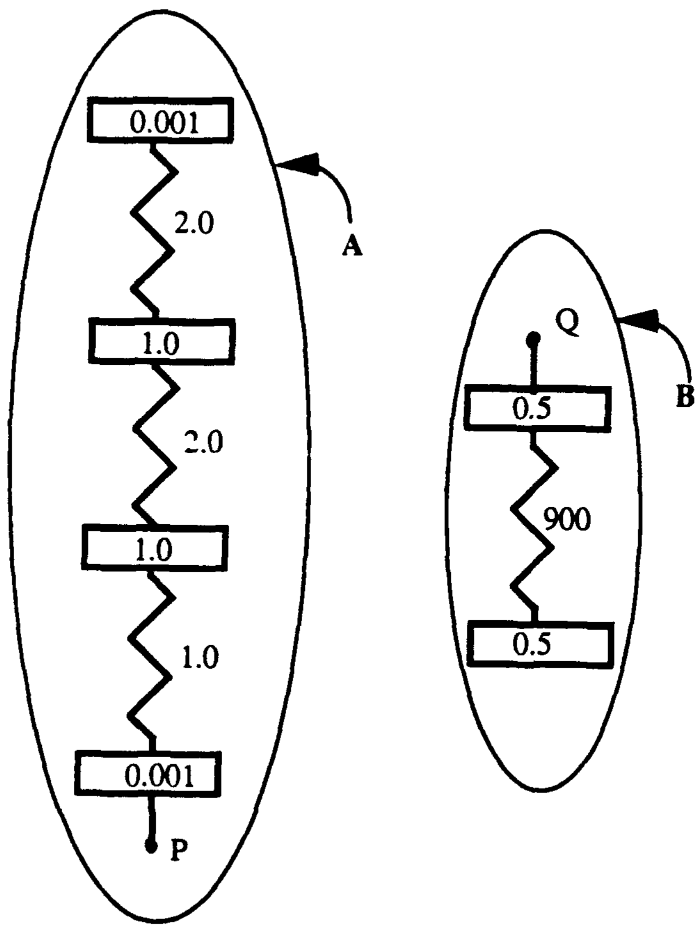

Fig. 2 Mass-Spring Examples

For A, the mass and stiffness matrices are respectively

$$M_A = \begin{bmatrix} 0.001 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0.001 \end{bmatrix} \tag{25}$$

$$K_A = \begin{bmatrix} 2.0 & -2.0 & 0 & 0 \\ -2.0 & 4.0 & -2.0 & 0 \\ 0 & -2.0 & 3.0 & -1.0 \\ 0 & 0 & -1.0 & 1.0 \end{bmatrix} \tag{26}$$

The eigenvalue and modal matrices are

$$\Lambda_A = \begin{bmatrix} 2002 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1001 \end{bmatrix} \tag{27}$$

and

$$\phi_A = \begin{bmatrix} 1.0000 & 0.5002 & 0.5000 & 0.0000 \\ -0.0010 & 0.4992 & 0.5000 & 0.0000 \\ 0.0000 & -0.4992 & 0.5000 & -0.0010 \\ 0.0000 & -0.5013 & 0.5000 & 1.0000 \end{bmatrix} \tag{28}$$

The corresponding matrices for B are:

$$M_B = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \tag{29}$$

$$K_B = \begin{bmatrix} 900 & -900 \\ -900 & 900 \end{bmatrix} \tag{30}$$

$$\Lambda_B = \begin{bmatrix} 0 & 0 \\ 0 & 3600 \end{bmatrix} \tag{31}$$

$$\phi_B = \begin{bmatrix} 0.7071 & -0.7071 \\ 0.7071 & 0.7071 \end{bmatrix} \tag{32}$$

If point P of A is now rigidly attached to point Q of B to form a combined system, the relevant matrices for the combined body are:

$$M = \begin{bmatrix} 0.001 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.5 \end{bmatrix} \tag{33}$$

$$K = \begin{bmatrix} 2.0 & -2.0 & 0 & 0 & 0 \\ -2.0 & 4.0 & -2.0 & 0 & 0 \\ 0 & -2.0 & 3.0 & -1.0 & 0 \\ 0 & 0 & -1.0 & 901.0 & -900.0 \\ 0 & 0 & 0 & -900.0 & 900.0 \end{bmatrix} \tag{34}$$

$$\Lambda = \begin{bmatrix} 2002 & 0 & 0 & 0 & 0 \\ 0 & 4.7 & 0 & 0 & 0 \\ 0 & 0 & 1.3 & 0 & 0 \\ 0 & 0 & 0 & 0.000 & 0 \\ 0 & 0 & 0 & 0 & 3597.4 \end{bmatrix} \tag{35}$$

$$\phi = \begin{bmatrix} 1.0 & 0.4925 & -0.4131 & -0.4472 & 0 \\ -0.001 & 0.4913 & -0.4129 & -0.4472 & 0 \\ 0 & -0.6718 & -0.1512 & -0.4472 & 0.0002 \\ 0 & 0.1796 & 0.5637 & -0.4472 & -0.7066 \\ 0 & 0.1801 & 0.5641 & -0.4472 & 0.7076 \end{bmatrix} \tag{36}$$

Suppose it is now decided to retain only two system modes – those with eigenvalues of 2002 and 4.7. The ϕ matrix is augmented as described earlier, then its last three columns are deleted, and the resulting matrix is partitioned in such a way as to generate the following two matrices:

$$\varphi_A = \begin{bmatrix} 1.0 & 0.4925 \\ -0.001 & 0.4913 \\ 0 & -0.6718 \\ 0 & 0.1796 \end{bmatrix} \tag{37}$$

$$\varphi_B = \begin{bmatrix} 0.0 & 0.1796 \\ 0.0 & 0.1801 \end{bmatrix} \tag{38}$$

These two matrices are then used as in Eqs.(17-24) to produce the following reduced eigenvalue matrices for A and B:

$$\lambda_A = \begin{bmatrix} 2002 & 0 \\ 0 & 5 \end{bmatrix} \tag{39}$$

and

$$\lambda_B = \begin{bmatrix} 0 & 0 \\ 0 & 3600 \end{bmatrix} \tag{40}$$

It is seen that λ_A and λ_B, as given in Eqs. (39) and (40), clearly approximate the original eigenvalue matrices of A and B respectively.

LARGE RELATIVE MOTION

The analyses presented above cease to be valid once any component of a system under consideration is allowed to take on large displacements relative to any other part of the system. Such is the case with the system shown in Fig. 2. This system consists of two flexible bodies A and B connected through a one degree of freedom hinge. If a multibody simulation code is to be used to study the dynamics of such a system, then, modal description of the model of each of the system components will be needed as input, while model reduction criteria will generally be available at the system level only. The model reduction technique described above can still be used if a quasi-static approach is adopted. This requires that system modal data be available for several key configurations of the system. For example, system modal data could be generated for six equally spaced orientations of the body B relative to the body A. The modal reduction technique described above can then be used to determine the needed component modes for each system configuration. It might seem that such consideration of many system configurations could give rise to a prohibitive number of required component modes. In practical cases, it is unlikely that the sets of required component modes that emerge from the system configurations considered will be disjoint; in fact, the difference between these sets should be practically null, so that the total number of component modes to be retained should remain reasonable.

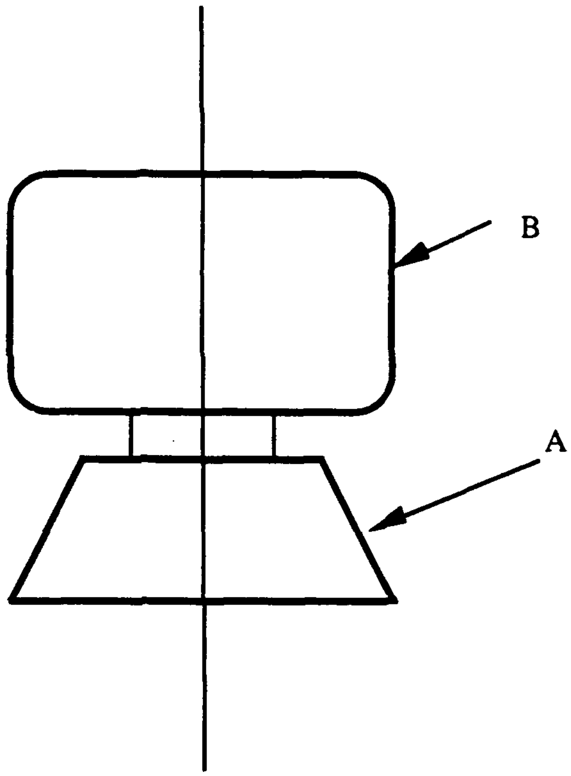

Fig. 3. System Permitting Large Relative Motion

SUMMARY AND CONCLUSION

The use of multibody simulation programs for the study of large displacement motions of systems of flexible bodies adds another dimension to the model reduction problem. In general, straightforward criteria are available for performing model reduction at the system level; however, simulation codes normally require modal data input for each component body of a system rather than for the system as a whole. It is thus always necessary to determine the modes to be retained for each component based on knowledge of the system modes of interest.

The method presented in this report for component model reduction utilizes submatrices of the system modal matrix as transformation matrices used to accomplish the first phase of a two phase matrix diagonalization process. This method is systematic and, as demonstratedabove using simple mass-spring systems, the method gives very accurate results. The method has also been tested on a model of the Galileo spacecraft, and gave excellent results.

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